Capítulo 5

1º) Comprobar que dada la acción:

$$S[\phi] = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

la ecuación de Schwinger-Dyson es:

$$m^2 Z'[J] + \frac{\lambda}{6}Z'''[J] = JZ[J]$$

$$Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi} \quad (1)$$

$$Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]+J\phi} \quad (2) \qquad Z'[J] = \int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]+J\phi}\,\phi \quad (3)$$

$$Z'''[J] = \int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]+J\phi}\,\phi^3 \quad (4) \qquad S'[\phi] = m^2\phi + \frac{\lambda}{6}\phi^3 \quad (5)$$

$$\int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]j+J\phi}\; S'[\phi] = \int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]j+J\phi}\left(m^2\phi + \frac{\lambda}{6}\phi^3\right) =$$

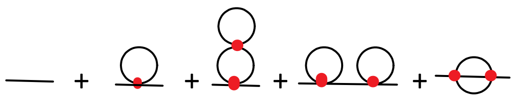
$$m^2 \underbrace{\int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]j+J\phi}\,\phi}_{Z'[J]} + \frac{\lambda}{6}\underbrace{\int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]j+J\phi}\phi^3}_{Z'''[J]} = -\underbrace{\int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]+J\phi}\left(-S'[\phi]+J\right)}_{\left[e^{-S[\phi]+J\phi}\right]_{-\infty}^{\infty} = 0} + \underbrace{\int_{-\infty}^{\infty} d\phi\, e^{-S[\phi]+J\phi}}_{Z[J]} J$$

$$m^2 Z'[J] + \frac{\lambda}{6}Z'''[J] = JZ[J]$$

2º) Calcular $<\phi^2>$ a segundo orden

a) Con los Diagramas de Feynman

b) Cálculo directo

a) $<\phi^2> =$ 

La figura con simetría más complicada de ver es la última, podemos ver que entre los dos puntos hay 3 líneas, entonces serían las permutaciones de ellas, es decir, 3!.

$$<\phi^2> \approx \frac{1}{m^2} + \frac{-\lambda}{2}\left(\frac{1}{m^2}\right)^3 + \frac{(-\lambda)^2}{2\cdot 2}\left(\frac{1}{m^2}\right)^5 + \frac{(-\lambda)^2}{2\cdot 2}\left(\frac{1}{m^2}\right)^5 + \frac{(-\lambda)^2}{3!}\left(\frac{1}{m^2}\right)^5$$

$$<\phi^2> \approx \frac{1}{m^2} - \frac{\lambda}{2m^6} + \frac{2\lambda^2}{3m^{10}}$$

b) $Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi} = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 + J\phi}\, \underbrace{e^{-\frac{\lambda}{24}\phi^4}}_{P.Taylor}$

$$Z[J] \approx \int_{-\infty}^{\infty} d\phi \; e^{-\frac{m^2}{2}\phi^2 + J\phi} \left(1 - \frac{\lambda}{24}\phi^4 + \frac{1}{2}\left(-\frac{\lambda}{24}\phi^4\right)^2\right)$$

$$Z[J] \approx \underbrace{\int_{-\infty}^{\infty} d\phi \; e^{-\frac{m^2}{2}\phi^2 + J\phi}}_{Z_0[J]} \left(1 - \frac{\lambda}{24}\phi^4 + \frac{\lambda^2}{2\cdot 24^2}\phi^8\right)$$

$$Z[J] \approx Z_0[J] - \frac{\lambda}{24} Z_0^{IV}[J] + \frac{\lambda^2}{2\cdot 24^2} Z_0^{VIII}[J]$$

$$Z[0] \approx Z_0[0] \left(1 - \frac{\frac{\lambda}{24} Z_0^{IV}[J]}{Z_0[0]} + \frac{\frac{\lambda^2}{2\cdot 24^2} Z_0^{VIII}[J]}{Z_0[0]}\right)$$

$$Z[0] \approx Z_0[0] \left(1 - \frac{\lambda}{24} < \phi^4 >_0 + \frac{\lambda^2}{2\cdot 24^2} < \phi^8 >_0\right)$$

$$Z[0] \approx Z_0[0] \left(1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{2\cdot 24^2}\frac{7\cdot 5\cdot 3}{m^8}\right) \quad (1)$$

$$Z''[0] \approx Z_0''[0] - \frac{\lambda}{24} Z_0^{VI}[0] + \frac{\lambda^2}{2\cdot 24^2} Z_0^{x}[0]$$

$$Z''[0] \approx Z_0[0] \frac{Z_0''[0] - \frac{\lambda}{24} Z_0^{VI}[0] + \frac{\lambda^2}{2\cdot 24^2} Z_0^{x}[0]}{Z_0[0]} \quad (2)$$

$$< \phi^2 > = \frac{Z''[0]}{Z[0]} \approx \frac{\dfrac{Z_0''[0] - \frac{\lambda}{24} Z_0^{VI}[0] + \frac{\lambda^2}{2\cdot 24^2} Z_0^{x}[0]}{Z_0[0]}}{1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{2\cdot 24^2}\frac{7\cdot 5\cdot 3}{m^8}}$$

$$< \phi^2 > \approx \frac{\frac{1}{m^2} - \frac{\lambda}{24}\frac{5\cdot 3}{m^6} + \frac{\lambda^2}{2\cdot 24^2}\frac{9\cdot 7\cdot 5\cdot 3}{m^{10}}}{1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{2\cdot 24^2}\frac{7\cdot 5\cdot 3}{m^8}} \approx f(\lambda) \; (P.Tailor\ en\ \lambda = 0)$$

$$f(0) = \frac{1}{m^2}$$

$$f'(0) = \left| \frac{-\frac{5\cdot 3}{24m^6} + \frac{\lambda}{24^2}\frac{9\cdot 7\cdot 5\cdot 3}{m^{10}}}{1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{2\cdot 24^2}\frac{7\cdot 5\cdot 3}{m^8}} + \frac{\left(\frac{1}{8m^4} - \frac{\lambda}{24^2}\frac{7\cdot 5\cdot 3}{m^8}\right)\left(\frac{1}{m^2} - \frac{\lambda}{24}\frac{5\cdot 3}{m^6} + \frac{\lambda^2}{2\cdot 24^2}\frac{9\cdot 7\cdot 5\cdot 3}{m^{10}}\right)}{\left(1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{2\cdot 24^2}\frac{7\cdot 5\cdot 3}{m^8}\right)^2} \right|_{\lambda=0}$$

$$f'(0) = -\frac{15}{24m^6} + \frac{1}{8m^4}\cdot\frac{1}{m^2} = -\frac{1}{2m^6}$$

$$f''(0)=\frac{9\cdot7\cdot5\cdot3}{24^2m^{10}}-\frac{1}{8m^4}\frac{5\cdot3}{24m^6}-\frac{7\cdot5\cdot3}{24^2m^8}\cdot\frac{1}{m^2}-\frac{5\cdot3}{24m^6}\frac{1}{8m^4}+\frac{1}{4m^4}\cdot\frac{1}{8m^4}\cdot\frac{1}{m^2}$$

$$f''(0)=\frac{945-105}{24^2m^{10}}-\frac{30}{192m^{10}}+\frac{1}{32m^{10}}=\frac{140}{96m^{10}}-\frac{15}{96m^{10}}+\frac{3}{96m^{10}}=\frac{128}{96m^{10}}=\frac{4}{3m^{10}}$$

$$<\phi^2>\approx f(0)+f'(0)\lambda+\frac{1}{2!}f''(0)\lambda^2=\frac{1}{m^2}-\frac{\lambda}{2m^6}+\frac{1}{2}\cdot\frac{4\lambda^2}{3m^{10}}=\frac{1}{m^2}-\frac{\lambda}{2m^6}+\frac{2\lambda^2}{3m^{10}}$$